Exercise 3

Vertices $P_1 = (1,2)$, $P_2 = (2,3)$
and P_3 $(4,1)$

F_5^2 over the field F_5 with 5 elements

$P_i P_j$ divided by t_{ij}

$T_{12} = 1$, $T_{23} = 2$ and $T_{31} = 3$

$P_1 t_{23}$, $P_2 t_{31}$ and $P_3 t_{12}$ concurrent?
If yes determine their intersection?

Solution

Assume that the vertices of the triangle ha
the coordinates

$P_1 = (1,0,0)$ $P_2 = (0,1,0)$, P_3 $(0,0,1)$
in affine F_3. Then t_{ij} have coordinates

$$T_{12} = \frac{1}{1+T_{12}} (1, T_{12}, 0)$$

$$T_{23} = \frac{1}{1+T_{23}} (0, 1, T_{23})$$

$$T_{31} = \frac{1}{1+T_{31}} (T_{31}, 0, 1)$$

Exercise 3

Lines through t_{ij} and P_k are given as the intersection of the triangle plane $x_1 + x_2 + x_3 = 1$ with the planes

$T_{12}\ x_1 = x_2$

$T_{23}\ x_2 = x_3$

$T_{31}\ x_3 = x_1$

$T_{12} = 1\ T_{23} = 2\ T_{31} = 3$

$$T_{12} = \frac{1}{1+1}(1,1,0) = \frac{1}{2}(1,1,0)$$

$$T_{23} = \frac{1}{1+2}(0,1,2) = \frac{1}{3}(0,1,2)$$

$$T_{31} = \frac{1}{1+3}(3,1,0) = \frac{1}{4}(3,1,0)$$

Assume the 3 lines are concurrent. Therefore there is common intersection point (x_1, x_2, x_3) of the 4 planes and its coordinates satisfy all the 4 equation

Exercise 3

$x_i \neq 0$

$x_1 + x_2 + x_3 = 1$ multiply both sides b
$T_{12} T_{23} T_{31} = 1$ you get.

$T_{12} T_{23} T_{31} = 1$ it is not parallel because at least two points intersect in point q Assume the lines $P_1 t_{23}$ then q satisfy the equation $T_{23} x_2 = x_3$ and $P_2 T_{31} \to T_{31} x_3 = x_1$

$P_1 \to (1, 2)$ $T_{23} = 2$
$(1, 2)\, 2 \to 2x_2 = x_3$

$T_{12} T_{23} T_{31} = 1$ it satisfy the first equation $T_{12} x_1 = x_2$ and q lies on $P_3 t_{12}$. Therefore bijective affine map from affine plane $x_1 + x_2 + x_3 = 1$ to F^2